Quasi-classical approach to perpendicular transport through metallic magnetic superlattices

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Abstract. We present an extended quasi-classical theory of the perpendicular transport of electrons through magnetic multilayers. The two-point conductivity and the perpendicular resistivity are analytically studied by treating the spin-dependent scattering at interfaces as bulk asymmetric scattering within mixed interlayers. The results obtained are shown to be equivalent to those derived from the quantum method, indicating that there is a direct connection between the Boltzmann and the Kubo approaches.

There has been great interest in the giant magnetoresistance (GMR) of metallic magnetic multilayers composed of a ferromagnetic film alternated with a non-magnetic film. A large negative GMR for currents in the plane (CIPs) of the layers was found first in Fe/Cr multilayers [1, 2] and subsequently in many other magnetic multilayers. Recently, a giant MR has been observed also with the currents perpendicular to the plane (CPPs) of the layers in Co/Ag, Co/Cu and Fe/Cr multilayers [3, 4].

Intensive theoretical studies of the giant MR have been made [5–20]. Of these the two most influential theories are the quasi-classical method [5–10] based on the Boltzmann equation and the quantum approach [11–14] starting from the Kubo formula. Both of them attribute the GMR effect to the spin dependent scattering at interfaces and within ferromagnetic films. The quantum approach is regarded at present as a more complete theoretical description because it is suitable not only for the CIP MR in various magnetic multilayers [11, 12] but also for the CPP MR in infinite superlattices [13, 14]. Furthermore, its prediction that CPP MR⩾CIP MR has been verified by experiments [3, 4]. The quasi-classical approach has widely been applied to study the CIP MR problem by including the spin-dependent interface scattering phenomenologically in terms of transmission, reflection and scattering coefficients [5, 6]. The Boltzmann equation approach to the CIP MR has been recently developed by Valet and Fert [19], in which spin-flip relaxation and the spin accumulation effect are taken into account.

In the present work we have extended the quasi-classical approach to the CPP conductivity in magnetic multilayers such as Fe/Cr superlattices and derive its analytical result by treating interfaces as thin regions of mixed Fe and Cr. This treatment was first proposed by Johnson and Camley [7] to account simultaneously for both the CIP MR and the overall CIP resistivity of the structure. They assumed that there is bulk asymmetric scattering in the mixed regions instead of asymmetric scattering at the sharp interfaces.

In this extended quasi-classical model the bulk and interface scattering are treated in the same way because the spin-dependent scattering at the sharp interfaces has been replaced by the bulk scattering within the mixed interlayers. In the CPP case, as a result, there are spin-dependent interface resistances which come from the spin-dependent bulk scattering within the mixed regions. This origin of the interface resistances is completely different from the spin accumulation effect discussed in [19]. In order to compare the extended quasi-classical approach with the quantum model proposed by Zhang and Levy [13] and Levy and Zhang [14], the same assumption has been made that there is neither spin-flip scattering (i.e. independent spin-up and spin-down channels) nor a spin accumulation effect.

Let us consider a multilayer structure, e.g. a Fe/Cr superlattice, with a very large number of bilayers stacked along the z direction. Each bilayer is composed of an Fe film and a Cr film separated by a thin film of mixed Fe and Cr. For a uniform electric field applied along the z direction, because of the inhomogeneity of the layered structures, the internal field varies from one layer to the next so that the magnitude E of the total electric field perpendicular to the layers depends on the position in the z direction [13]. In this case the Boltzmann equation is given by

$$\frac{\partial g_s}{\partial z} + \frac{g_s}{\tau_s(z)v_z} = eE(z)\frac{\partial f_0}{\partial \varepsilon}. \tag{1}$$

Here f_0 is the equilibrium distribution function, $g_s(v, z)$ is the correction to the distribution function and $\tau_s(z)$ is the z-dependent relaxation time due to bulk scattering. Following Johnson and Camley [7], we set $\tau_s(z)$ equal to $\tau_s(\text{Fe})$ for z on the Fe films and to $\tau_s(\text{M})$ for z within the mixed regions at interfaces, both of them being spin dependent, and to be equal to $\tau(\text{Cr})$ (spin independent) for z on the Cr films. In the present model, since the effective interface scattering has been taken into account in the mixed regions, we can assume that electrons pass freely through the boundaries between the mixed region and the Fe or Cr film, at which there is neither reflection nor scattering. The differential equation (1) is exactly solvable. Its general solution is easily shown to have the following path-integral form:

$$g_s(v, z) = T_s(z, z')g_s(v, z') + \int_{z'}^{z} \mathrm{d}u \, T_s(z, u)eE(u)\frac{\partial f_0}{\partial \varepsilon} \tag{2}$$

with

$$T_s(z, z') = \exp\left(-\int_{z'}^{z} \mathrm{d}u \, [\tau_s(u)v_z]^{-1}\right). \tag{3}$$

For multilayers with a very large number of bilayers, the surface scattering has little effect on the resistivity. Thus, the boundary conditions at the two outer surfaces are of no importance; instead we use periodic boundary conditions, $\tau_s(z) = \tau_s(z + mL)$ with L the length of the sample in the z direction and m an arbitrary integer. It then follows that $g_s(v, z) = g_s(v, z - mL)$. Taking $z' = z - mL$ in equation (2) and using the periodic boundary condition for $g_s(v, z)$, one finds that

$$g_s(v, z) = [1 - T_s(z, z - md)]^{-1} \int_{z-mL}^{z} \mathrm{d}z' \, T_s(z, z')eE(z')\frac{\partial f_0}{\partial \varepsilon}. \tag{4}$$

It is easy to see from equation (3) that $\lim_{mL\to\infty}[T_s(z, z - mL)] = 0$ for $v_z > 0$ and $\lim_{mL\to-\infty}[T_s(z, z - mL)] = 0$ for $v_z < 0$. By the use of the two limits, we obtain

$$g_s(v, z) = e\frac{\partial f_0}{\partial \varepsilon}\left(\Theta(v_z)\int_{-\infty}^{z} \mathrm{d}z' \, T_s(z, z')E(z') - \Theta(-v_z)\int_{z}^{\infty} \mathrm{d}z' \, T_s(z, z')E(z')\right) \tag{5}$$

with $\Theta(x)$ the unit step function. It is convenient to introduce a diagonal two-point conductivity tensor [20] to describe the electrical linear response of the system to an external electric field. Its CPP component $\sigma_s^\perp(z, z')$ is defined by

$$J_s^\perp(z) = \int_{-\infty}^{\infty} dz' \, \sigma_s^\perp(z, z') E(z') \tag{6}$$

where $J_s^\perp(z)$ is the spin- and z-dependent current density along the z direction. On the other hand, the CPP current density can be obtained by averaging over the electron distribution:

$$J_s^\perp(z) = em^3 \int d^3v \, g_s(v, z) v_z. \tag{7}$$

Substituting equation (5) into equation (7) and then comparing it with equation (6), one finds that

$$\sigma_s^\perp(z, z') = e^2 m^3 \int d^3v \, T_s(z, z') \frac{\partial f_0}{\partial \varepsilon} v_z[\Theta(v_z)\Theta(z - z') - \Theta(-v_z)\Theta(z' - z)]. \tag{8}$$

This integral is evaluated to give

$$\sigma_s^\perp(z, z') = (3C_D/4) E_3(|\chi_s(z) - \chi_s(z')|) \tag{9}$$

$$\chi_s(z) = \int_0^z \frac{du}{\lambda_s(u)}. \tag{10}$$

Here $\lambda_s(u) = v_F \tau_s(u)$ is the mean free path which depends not only on spin but also on position in the z direction with v_F the Fermi velocity. $C_D = ne^2/mv_F$ with n the density of conduction electrons [20]. $E_m(x) = \int_1^\infty dt \exp(-xt)/t^m$ is the exponential integral function of order m. Equation (9) is one of the major results of this work. It is straightforward to show that the present result for $\sigma_s^\perp(z, z')$ derived from the extended quasi-classical approach is identical with that obtained from the Kubo formula in real space (equation (17) of [20]).

    After obtaining the CPP component of the two-point conductivity tensor, we now calculate the CPP conductivity. Since spin-flip scattering is neglected in the present model, each spin direction contributes to the current independently and the total conductivity is the sum from each spin direction. From the equation of continuity for current, one finds that $\partial J_s^\perp(z)/\partial z = 0$. This implies that $J_s^\perp(z)$ is a constant independent of z. However, $E(z)$ is not uniform, having different values for the films of Fe and Cr as well as the mixed region. Thus, the CPP conductivity is defined as $\sigma_\perp = (\sum_s J_s^\perp)/\bar{E}$ where $\bar{E} = \int_0^L E(z) \, dz/L$ is the average electric field over the length of the sample in the z direction. By dividing both sides of equation (6) by $L\lambda_s(z)$ and then integrating over z within the whole system, one finds that

$$J_s^\perp/\bar{\lambda}_s = L^{-1} \int_0^L \frac{dz}{\lambda_s(z)} \int_{-\infty}^{\infty} dz' \, \sigma_s^\perp(z, z') E(z') \tag{11}$$

where

$$(\bar{\lambda}_s)^{-1} \equiv L^{-1} \int \frac{dz}{\lambda_s(z)} = L^{-1} \sum_i \frac{d_i}{\lambda_{is}} \tag{12}$$

is the inverse self-averaging free path with d_i the thickness of the ith layer and λ_{is} the mean free path for electrons with spin s in the ith layer. The above summation extends over all layers including the mixed interlayers. From the periodic boundary condition used above, it follows that there is a periodicity of the two-point conductivity: $\sigma_s^\perp(z, z') = \sigma_s^\perp(z + mL, z' + mL)$. If this is taken into account, the double integral on the right-hand side of equation (11) can be rewritten as

$$\frac{J_s^\perp}{\bar{\lambda}_s} = L^{-1} \int_0^L dz' E(z') \int_{-\infty}^\infty \frac{dz\, \sigma_s^\perp(z, z')}{\lambda_s(z)}. \tag{13}$$

With the aid of equations (9) and (10) the integral over z is given by

$$\frac{3C_D}{4} \int_1^\infty \frac{dt}{t^3} \int_{-\infty}^\infty d\chi_s(z) \exp[-t|\chi_s(z) - \chi_s(z')|] \tag{14}$$

which is easily evaluated to give $C_D/2$. It then follows from equations (11) and (12) that $J_s^\perp = C_D \bar{\lambda}_s \bar{E}/2$, and so the CPP resistivity is given by

$$\rho_s^\perp = \frac{2mv_F}{ne^2} L^{-1} \sum_s \frac{d_i}{\lambda_{is}} = L^{-1} \sum_s \rho_{is} d_i \tag{15}$$

where $\rho_{is} = 2mv_F/ne^2\lambda_{is}$ is the resistivity of the ith layer for the current channel with spin s. Equation (15) is a well known formula for resistances in series, i.e. the actual resistance for a current channel can be regarded as a series of resistance of length d_i. This conclusion coincides with that obtained by Zhang and Levy [13], indicating that there is a close connection between the quasi-classical and quantum approaches. The recent work of Valet and Fert [19] also shows that, for layer thicknesses much shorter than the spin diffusion length, the CPP resistance of a multilayer can be calculated by use of a simple series summation of resistances due to the ferromagnetic and non-magnetic layers plus 'interface resistances'. Since two current channels for spin-up and spin-down electrons are assumed to contribute independently, the total conductivity is the sum from each channel and the total resistivity is given by $\rho_\perp = 2mv_F/[ne^2(\bar{\lambda}_+ + \bar{\lambda}_-)]$ where $\bar{\lambda}_+(\bar{\lambda}_-)$ is the self-averaging free path for the spin-up (spin-down) channel. The formula for a network of resistors connected in series has already been used for the interpretation of experimental results on Ag/Co and AgSn/Co multilayers [3].

In what follows we wish to compare the present result for $\bar{\lambda}_s$ with that derived from the quantum approach [14]. Taking an example, we consider an Fe/Cr periodic superlattice with an Fe film of thickness a and a Cr film of thickness b separated by a mixed interlayer of thickness d. The mean free paths in three types of film are $\lambda(Cr) = v_F\tau(Cr)$, $\lambda_s(Fe) = v_F\tau_s(Fe)$ and $\lambda_s(M) = v_F\tau_s(M)$. For the ferromagnetic configuration the spin-dependent $\lambda_s(Fe)$ and $\lambda_s(M)$ remain unchanged in all the Fe films and the mixed interlayers. With the aid of these parameters, equation (12) reduces to

$$\frac{1}{\bar{\lambda}_s} = \frac{1}{a + b + 2d} \left(\frac{a}{\lambda_s(Fe)} + \frac{b}{\lambda(Cr)} + \frac{2d}{\lambda_s(M)} \right) \tag{16}$$

where the factor 2 comes from the fact there are two Fe–Cr interfaces adjacent to each Fe film. For the antiferromagnetic configuration the period of the magnetic unit cell is $2(a + b + 2d)$ and equation (12) becomes

$$\frac{1}{\bar{\lambda}_s} = \frac{1}{2(a + b + 2d)} \sum_s \left(\frac{a}{\lambda_s(Fe)} + \frac{b}{\lambda(Cr)} + \frac{2d}{\lambda_s(M)} \right). \tag{17}$$

Here the summation over s includes $s = +$ and $-$ because the two adjacent Fe films in a magnetic unit cell have opposite magnetization directions. In order to connect equations (16) and (17) with equation (20) of [14] the mean free paths defined above are taken as

$$\lambda(Cr) = \lambda_b$$

$$\lambda_s(Fe) = \lambda_b/(1 + p^2 + 2p\xi_s) \tag{18}$$

$$\lambda_s(M)/d = \lambda_s'/(1 + p^2 + 2p\xi_s)$$

with $\xi_+ = 1$ and $\xi_- = -1$. All the parameters on the right-hand side of the above equations were defined in [14], in which p stands for the ratio of spin-dependent to spin-independent scattering for both the interfaces and the bulk. Substitution of equation (18) into equations (16) and (17) brings us to the result for the inverse self-averaged free path

$$(\bar{\lambda}_s)^{-1} = \bar{\lambda}_s(b)^{-1} + \bar{\lambda}_s(M)^{-1} \tag{19}$$

with

$$\bar{\lambda}_s(b) = \lambda_b(a + b + 2d)/[a + b + a(p^2 + 2p\xi_s \cos\theta)] \tag{20}$$

$$\bar{\lambda}_s(M) = \lambda_s'(a + b + 2d)/[2(1 + p^2 + 2p\xi_s \cos\theta)] \tag{21}$$

where $\theta = 0$ for the ferromagnetic configuration and $\theta = \pi/2$ for the antiferromagnetic configuration. Since d is much smaller than $a + b$, we have $a + b + 2d \simeq a + b$. Under this approximation, equations (19)–(21) are found to be just the same as equation (20) of [14]. This means that, starting from the Boltzmann equation, one can obtain the same result for the CPP resistivity as that derived from the Kubo formula as long as the spin-dependent scattering at the interfaces is properly treated.

The present theory is not confined to periodic infinite superlattices. It is also suitable for non-periodic or finite multilayers provided that L, the length of the sample in the z direction, is much greater than $\bar{\lambda}_s$. If the condition $L \gg \bar{\lambda}_s$ is not satisfied, the present result becomes invalid. In this case, one should replace the outgoing boundary conditions by the Dirichlet boundary condition [19]. It was proposed [16] that the Landauer–Buttiker scattering formalism can be used to calculate the CPP resistance of microstructured multilayers.

Summarizing, in this work we develop an extended quasi-classical approach to the CPP transport in magnetic multilayers on condition that $L \gg \bar{\lambda}_s$. Explicit expressions for the CPP components of the two-point conductivity tensor and the self-averaged free paths have been derived. They are shown to be equivalent to those obtained from the Kubo formula.

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